

LINEAR SYSTEMS AND ROW REDUCTION

For each of the linear systems below, form the augmented matrix and find its row-reduced echelon form. State whether the system has zero, one, or infinite solutions.

(a)	(b)	(c)	(d)
$x_1 + x_2 + x_3 = 0$ $x_1 + x_2 = 3$ $x_2 + x_3 = 1$	$x_1 + x_2 + 2x_3 = -1$ $x_1 - 2x_2 + x_3 = -5$ $3x_1 + x_2 + x_3 = 3$	$x_1 + 2x_2 + 3x_3 = 0$ $x_1 + x_2 + x_3 = 0$ $x_1 + x_2 + 2x_3 = 0$ $x_1 + 3x_2 + 3x_3 = 0$	$x_1 + 2x_2 + 3x_3 = 0$ $x_1 + x_2 + x_3 = 0$ $5x_1 + 7x_2 + 9x_3 = 0$

LINEAR REGRESSION

We have seen how we can use linear regression to find the best-fit line for a set of data points by solving the equations

$$X^T X a = X^T y$$

This is a system of linear equations known as the “normal equations.” We also used MATLAB to fit higher-order models such as quadratic functions, or quartic functions. The same technique can be used to find 2-D models of the form

$$\hat{z} = a_1 x + a_2 y + a_3$$

Given the following data points (x, y, z) :

x	y	z
$x_1 = 1$	$y_1 = 1$	4
$x_2 = 1$	$y_2 = 3$	0
$x_3 = 2$	$y_3 = 1$	6
$x_4 = 2$	$y_4 = 3$	3

- (a) Using the values of x and y , construct a matrix X such that $\hat{z} = X\vec{a}$, where $\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$. In other

words: $Xa = \begin{bmatrix} a_1 x_1 + a_2 y_1 + a_3 \\ a_1 x_2 + a_2 y_2 + a_3 \\ \dots \end{bmatrix}$

- (b) Setup the augmented matrix for the system of equations $X^T X \vec{a} = X^T \vec{z}$ and solve for \vec{a} using row reduction.

LINEAR TRANSFORMATIONS

The four matrices below define four linear transformations.

Scale	Shear	Rotation	Translation
$U = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$H = \begin{bmatrix} 1 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$R = \begin{bmatrix} \cos \pi/2 & -\sin \pi/2 & 0 \\ \sin \pi/2 & \cos \pi/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$T = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

The original shape, denoted S_0 is shown on the next page. Match the following linear transformations with its image on the next page.

1. $S = TRS_0$

2. $S = HRS_0$

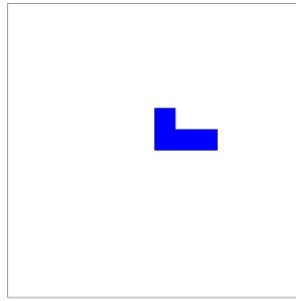
3. $S = RRS_0$

4. $S = RTS_0$

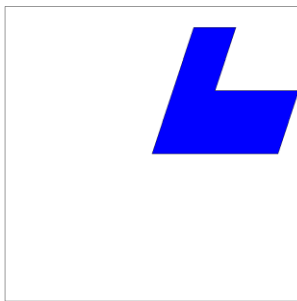
5. $S = UHS_0$

6. $S = RUS_0$

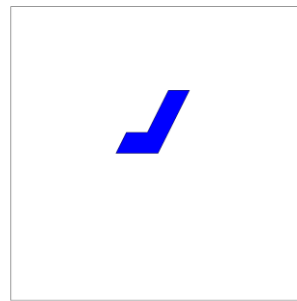
Shapes



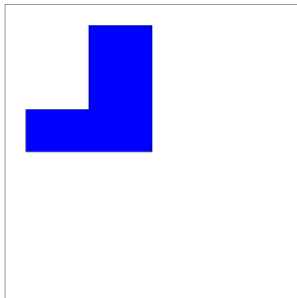
(original)



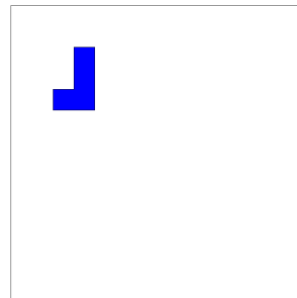
(a)



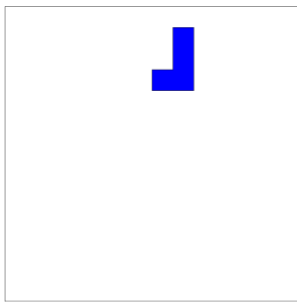
(b)



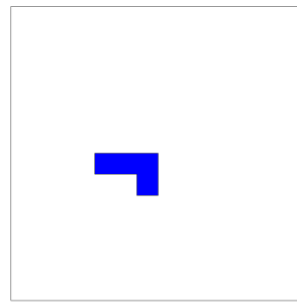
(c)



(d)



(e)



(f)